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# Characteristic dimensions and prediction at high frequencies of the surface impedance of porous layers

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The surface acoustic impedance of a glass wool and a reticulated foam is measured in a free field up to 20 000 Hz. The characteristic dimensions  $\Lambda$  and  $\Lambda'$  can be calculated for the glass wool, and the surface impedance can be predicted with no adjustable parameters. The motion of the frame is not taken into account. The agreement between measurement and prediction is good. For the foam, the characteristic dimensions cannot be calculated, because the geometry of the frame is not simple. A correct choice of  $\Lambda$  and  $\Lambda'$  allows a precise prediction of the surface impedance for a large range of frequencies and different thicknesses.

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## INTRODUCTION

In an important paper, Johnson *et al.*<sup>1</sup> gave a simple expression for the effective density  $\rho$  of a fluid saturating a porous medium, which simultaneously takes into account the inertial and the viscous forces, and presents a correct asymptotic behavior at high frequencies. As indicated by Johnson, the exact expression for the effective density cannot be calculated for the case of porous materials with a frame having a complicated geometry, but the asymptotic high-frequency behavior can be predicted if the tortuosity  $\alpha_\infty$ , and the characteristic viscous dimension  $\Lambda$ , are known (the tortuosity was previously denoted by  $k_s$  by Zwikker and Kosten,<sup>2</sup> and  $q^2$  by Attenborough,<sup>3</sup> and can be evaluated from electrical conduction measurements<sup>4,5</sup>).

The characteristic dimension  $\Lambda$  is given by<sup>1</sup>

$$\frac{2}{\Lambda} = \frac{\int_S |v^2(M)| dS}{\int_V |v^2(M)| dV}, \quad (1)$$

where  $v(M)$  is the microscopic velocity field related to the steady flow of an inviscid fluid in the material, and the integrals in the numerator and the denominator are performed over the contact surface between the fluid and the frame, and the fluid volume, respectively.

For the case of materials having parallel cylindrical pores,  $\Lambda$  is two times as large as the ratio of the area and the perimeter of the cross section, and is equal to the radius of the pores if they are circular cross-sectional shaped. In a similar way, Champoux and Allard<sup>6</sup> have worked out a simple expression for the bulk modulus  $K$  of the air in a porous medium. This expression presents a correct asymptotic behavior at high frequencies, where  $K$  mainly depends on a thermal characteristic dimension  $\Lambda'$  given by

$$\frac{2}{\Lambda'} = \frac{\int_S dS}{\int_V dV}. \quad (2)$$

Both Eqs. (1) and (2) are identical when the square velocities are removed. For the case of identical cylindrical pores,  $v(M)$  is a constant that can be removed from Eq. (1) and the two characteristic dimensions are equal. If the pores do not have a constant cross section, the dimension  $\Lambda$  is smaller than  $\Lambda'$ , because the weighting by the square velocity gives a larger contribution of the constrictions of the pores in Eq. (1).

## I. CHARACTERISTIC DIMENSIONS FOR FIBROUS MATERIALS

Fibrous materials such as glass wool are anisotropic because generally most of the fibers lie in planar planes parallel to the surface of the samples. A layer is represented in Fig. 1. The direction normal to the surface is  $x_3$  and planar planes are parallel to the  $x_1x_2$  plane. The characteristic dimensions are different for waves propagating parallel and perpendicular to the planar planes and these different characteristic dimensions must be known to predict the surface impedance of the layer at oblique incidence. It was shown by Allard and Champoux<sup>7</sup> that the characteristic dimensions could be calculated for the case of glass wools, the fibers being modeled as circular cross-sectional-shaped cylinders of total length  $L$  per unit volume, and of radius  $R$ , lying in planar planes. The dimensions  $\Lambda_N$  and  $\Lambda'_N$  related to propagation in the normal direction are given by<sup>7</sup>

$$\Lambda_N = 1/2\pi LR, \quad \Lambda'_N = 2\Lambda_N. \quad (3)$$

If the fibers are parallel in the planar direction, two orthogonal directions parallel to the planar planes can be defined,  $x_1$  parallel to the fibers and  $x_2$  perpendicular to the fibers. For these two directions, the characteristic dimensions  $\Lambda_1$ ,  $\Lambda'_1$ ,  $\Lambda_2$ , and  $\Lambda'_2$  are given by<sup>7</sup>

$$\Lambda_1 = \Lambda'_1 = 2\Lambda_N, \quad \Lambda_2 = \Lambda_N, \quad \Lambda'_2 = 2\Lambda_N. \quad (4)$$

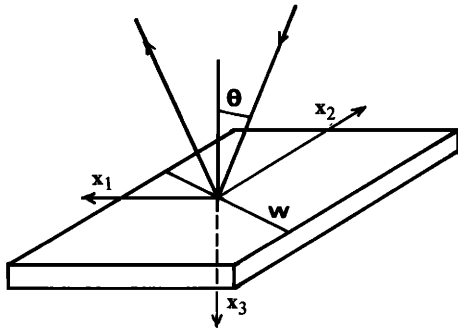


FIG. 1. The material backed by a rigid impervious layer. A plane wave in air impinges upon the material under an angle of incidence  $\theta$ . The axis  $w$  is the intersection of the incidence plane with the surface of the layer.

If the directions of the fibers are equi-distributed in the planar planes, there is only one dimension  $\Lambda_p$  and one dimension  $\Lambda'_p$  related to propagation parallel to the planar plane ( $x_1, x_2$  plane), regardless the orientation of axes  $x_1$  and  $x_2$ . They are given by

$$\Lambda_p = 4\Lambda_N/3, \quad \Lambda'_p = \Lambda'_N. \quad (5)$$

The demonstration of Eq. (5) is given in the Appendix.

## II. PREDICTION OF THE SURFACE IMPEDANCE

### A. Normal incidence

The expression for the effective density of the air in a porous material obtained from Eq. (3.4) of Ref. 1 is

$$\rho = \alpha_\infty \left[ \rho_0 + \frac{\sigma\phi}{j\omega\alpha_\infty} \left( 1 + \frac{4j\alpha_\infty^2\eta\rho_0\omega}{\sigma^2\Lambda_N^2\phi^2} \right)^{1/2} \right], \quad (6)$$

where  $\rho_0$  is the density and  $\eta$  the viscosity of air, and  $\alpha_\infty$ ,  $\sigma$ , and  $\phi$  are the tortuosity, the flow resistivity, and the porosity of the material. [The chosen time dependence is  $\exp(j\omega t)$ .] This expression presents the correct asymptotic behavior when  $\omega$  tend to infinity,<sup>1</sup>

$$\rho = \alpha_\infty [1 + (1-j)(\delta/\Lambda_N)]\rho_0, \quad (7)$$

where  $\delta$  is the viscous skin depth,

$$\delta = (2\eta/\rho_0\omega)^{1/2}. \quad (8)$$

It may be noticed that when  $\omega$  tends to zero, the imaginary part of  $\rho$  tends to the correct limit  $\sigma\phi/j\omega$ . We use the following expression for the bulk modulus of the air in the material:

$$K = \gamma P_0 \left[ \gamma - (\gamma - 1) \left[ 1 + \frac{8\eta}{j\Lambda_N^2 B^2 \omega \rho_0} \times \left( 1 + j\rho_0 \frac{\omega B^2 \Lambda_N^2}{16\eta} \right)^{1/2} \right]^{-1} \right]^{-1}, \quad (9)$$

where  $\gamma$  and  $B^2$  are the ratio of the specific heats and the Prandtl number of air, and  $P_0$  is the air equilibrium absolute pressure.

When  $\omega$  tends to infinity, Eq. (9) can be rewritten

$$K = \gamma P_0 [1 + (\gamma - 1)(1 - j)(\delta/B\Lambda'_N)]^{-1}, \quad (10)$$

which gives the correct asymptotic behavior for the bulk modulus.<sup>6</sup> At low frequencies  $K$  tends to  $P_0$  which is the isothermal bulk modulus. The complex wave number  $k$  in the porous material is given by

$$k = \omega(\rho/K)^{1/2}. \quad (11)$$

At high frequencies, at the first-order approximation,  $k$  is given by

$$k = \omega \left( \frac{\rho_0}{\gamma P_0} \right)^{1/2} \alpha_\infty^{1/2} \left[ 1 + (1-j) \frac{\delta}{2} \left( \frac{1}{\Lambda_N} + \frac{\gamma-1}{B\Lambda'_N} \right) \right], \quad (12)$$

where  $(\gamma-1)/B$  is equal to 0.47 for air in normal conditions. The surface impedance  $Z$  at normal incidence of a layer backed by a rigid surface is given by

$$Z = (Z_c/\phi) \cotan kl, \quad (13)$$

where  $l$  is the thickness of the layer and  $Z_c$  is the characteristic impedance,

$$Z_c = (\rho K)^{1/2}. \quad (14)$$

### B. Oblique incidence

At oblique incidence, the possible anisotropy of the material must be taken into account. We only consider the case of materials such as  $k$  and  $Z_c$  are the same in all directions parallel to the planar planes. The effective density  $\rho$  and the bulk modulus  $K$  are represented by tensors that have the same principal axes, two perpendicular axes  $x_1$  and  $x_2$  in the planar plane and  $x_3$ . Then, it is possible to define two complex wave numbers  $k_N$  and  $k_p$  and two characteristic impedances  $Z_c^N$  and  $Z_c^p$  in the normal and in all the planar directions, by using Eqs. (6), (9), (11), and (14), with the appropriate values for  $\Lambda$  and  $\Lambda'$ . Sound propagation in these materials is completely characterized by the couples  $k_p, Z_c^p$  and  $k_N, Z_c^N$ . The porous layer is represented in Fig. 1 in a plane, oblique acoustic field. Let  $\theta$  be the angle of incidence. Let  $k_0$  be the wave number in air.

The surface impedance is given by<sup>8,9</sup>

$$Z = \frac{(\rho_N K_N)^{1/2} k_N}{\phi k_3} (-j \cotan k_3 l), \quad (15)$$

where  $k_3$  is the  $x_3$  component of the wave number vector in the material

$$k_3 = k_N (1 - k_0^2 \sin^2 \theta / k_p^2)^{1/2}. \quad (16)$$

## III. HIGH-FREQUENCY MEASUREMENT OF SURFACE IMPEDANCE

At high frequencies, the Kundt tube method is not convenient, because the diameter of the tube must be very small, and it is difficult to prepare homogeneous samples of material for the measurement. We have used a free-field method<sup>10,11</sup> which is suitable at high frequencies. The experimental setup is shown in Fig. 2.

Two small electret microphones are used. The distance between both microphones is about 0.6 cm, and the source located at point S is set 2 m from the porous material. At high frequencies the acoustic field around the microphones

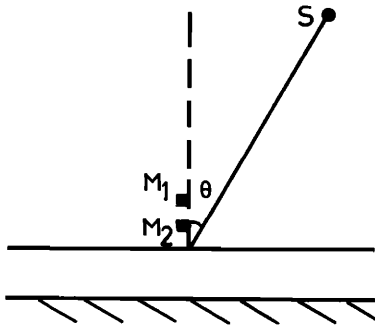


FIG. 2. The experimental setup for the measurement of the impedance. The porous material backed by a rigid and impervious layer, the two microphones at  $M_1$  and  $M_2$ , and the source  $S$ .

can be approximated with a good precision by a plane field. The reflection coefficient can be expressed as a function of the ratio of the pressures  $p_1$  and  $p_2$  at  $M_1$  and  $M_2$  as follows:<sup>11</sup>

$$R(\theta) = \frac{\exp(-ik_0d_2 \cos \theta) - \exp(-ik_0d_1 \cos \theta)p_2/p_1}{\exp(ik_0d_1 \cos \theta)p_2/p_1 - \exp(ik_0d_2 \cos \theta)} \quad (17)$$

In this equation,  $d_1$  and  $d_2$  are the distances from  $M_1$  and  $M_2$  to the layer, and  $k_0$  is the wave number in air. The surface impedance is given by

$$Z(\theta) = [R(\theta) + 1] / [\cos \theta \{-R(\theta) + 1\}] \quad (18)$$

#### IV. MEASUREMENT AND PREDICTION OF SURFACE IMPEDANCE FOR TWO MATERIALS

Two materials were studied, a glass wool and a reticulated plastic foam. The measured parameters  $\sigma$ ,  $\phi$ , and  $\alpha_\infty$ , for both materials, are given in Table I. The flow resistivity of the glass wool was measured for ten samples, and the measured values were located between 4200 and 5200  $\text{N m}^{-4} \text{ s}$ . The chosen value, 5000  $\text{N m}^{-4} \text{ s}$  gives the best agreement between measurement and prediction of the surface impedance at normal incidence. The thickness of the layer of glass wool is equal to 14 cm and two layers of foam, of thicknesses 4.1 and 6.15 cm have been used. The density  $\rho_1$  of the glass wool is equal to 6  $\text{kg/m}^3$ . An empirical law<sup>12</sup> can be used to calculate the average radius of the fibers:

$$R = \frac{1}{2} \left( \frac{3.18 \times 10^{-9}}{\sigma} \rho_1^{1.53} \right)^{1/2} \quad (19)$$

and  $L$  in Eqs. (3) is given by

$$L = \rho_1 / (\pi R^2 \rho_m) \quad (20)$$

where  $\rho_m$  is the density of glass, equal to  $2.5 \times 10^3 \text{ kg/m}^3$ .

TABLE I. The measured parameters for both materials.

	Flow resistivity, $\sigma$ $\text{N m}^{-4} \text{ s}$	Porosity, $\phi$	Tortuosity, $\alpha_\infty$
Glass wool	5000	0.99	1
Foam	7000	0.98	1.08

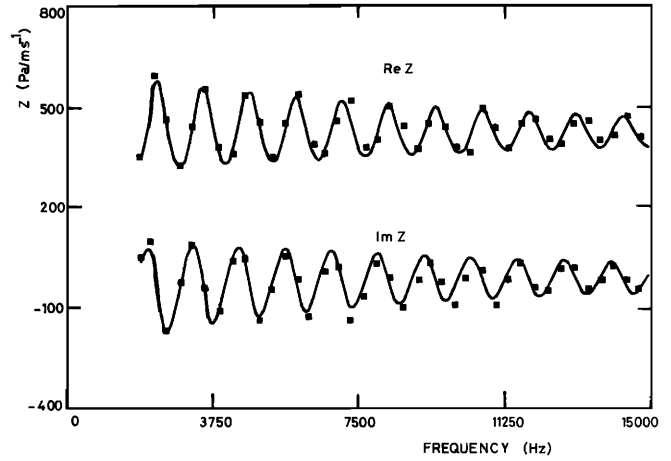


FIG. 3. The impedance  $Z$  of the layer of glass wool at normal incidence. Prediction —; measurement ■■■.

The two characteristic dimensions  $\Lambda_N$  and  $\Lambda'_N$  are equal to  $\Lambda = 3.25 \times 10^{-4} \text{ m}$ ,  $\Lambda' = 6.5 \times 10^{-4} \text{ m}$ . The measured and predicted surface impedances of the layer of glass wool are represented in Fig. 3 for  $\theta = 0^\circ$  and in Fig. 4 for  $\theta = 30^\circ$ . For angles of incidence equal to or larger than  $45^\circ$  the free-field method of measuring the impedance does not work at high frequencies. The sample of glass wool is rectangular and has an area equal to  $1 \times 1.8 \text{ m}$ . Measurements were performed for different distances of the microphones from the edge of the sample. For distances larger than 25 cm, the results were very close to each other, the differences being always smaller than  $40 \text{ P/(m s}^{-1}\text{)}$ . This shows that the finite dimension effect is negligible when the microphones are far from the edge of the material. It also shows that the local fluctuations of the properties of the glass wool are averaged when the free-field method is used, due to the area of material involved in a measurement. The agreement between prediction and measurement is good at normal and oblique incidence. It is worth noting that no adjustable parameter was necessary, if the uncertainty in the flow resistivity is not taken into account. The characteristic dimensions are not calculable for the case of the

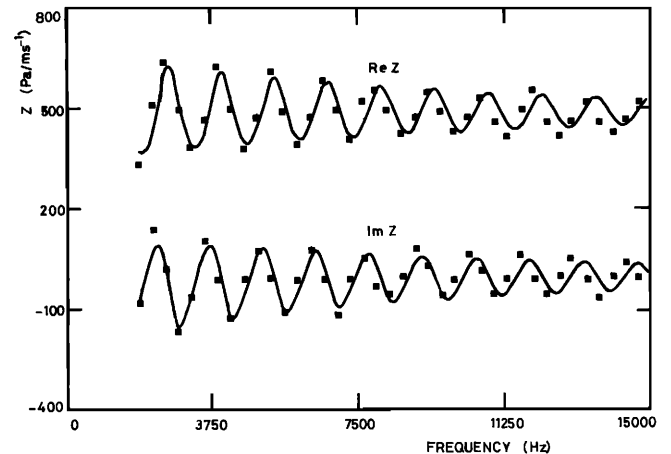


FIG. 4. The impedance  $Z$  of the layer of glass wool at an angle of incidence  $\theta = 30^\circ$ . Prediction —; measurement ■■■.

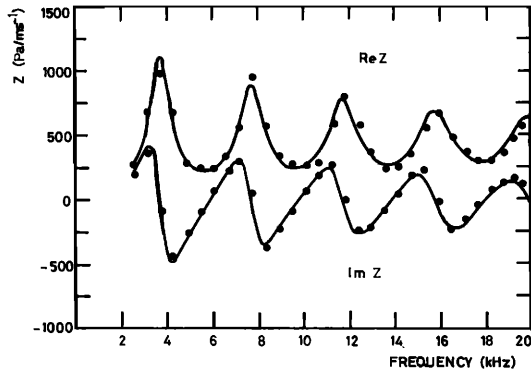


FIG. 5. The impedance  $Z$  of the layer of reticulated foam having a thickness equal to 4.1 cm at normal incidence. Prediction —; measurement ●●●.

foam, and we have tried to evaluate  $\Lambda$  and  $\Lambda'$  from acoustic measurements. The variations of the surface impedance at high frequencies are related to the interference between the incoming wave in the material and the wave reflected by the backing. The amplitude of the reflected wave depends on the damping in the material, which is given by the imaginary part of  $k$ :

$$\text{Im } k = -\omega \left( \frac{\rho_0}{P_0 \gamma} \right)^{1/2} \alpha_\infty^{1/2} \frac{\delta}{2} \left( \frac{1}{\Lambda} + \frac{\gamma-1}{B\Lambda'} \right). \quad (21)$$

It is not possible to separate the dependance of the damping on  $\Lambda$  from the dependance on  $\Lambda'$ . Nevertheless, due to the fact that  $\Lambda'$  is larger than  $\Lambda$ , and the presence of the coefficient  $(\gamma-1)/B$ , which reduces the dependence on  $\Lambda'$ , an order of magnitude of  $\Lambda$  can be obtained. For the second material, measured and predicted surface impedances of two layers of thickness 4.1 and 6.15 cm are represented in Figs. 5 and 6. The characteristic dimensions for the prediction are  $\Lambda = 1.7 \times 10^{-4}$  m, and  $\Lambda' = 3.9 \times 10^{-4}$  m. Johnson *et al.* in Ref. 1 pointed out that  $\Lambda$  must be of the same order of magnitude as  $[8\alpha_\infty \eta / (\sigma \phi)]^{1/2}$ . This quantity is equal to  $1.5 \times 10^{-4}$  m, very close to the evaluated value of  $\Lambda$ .

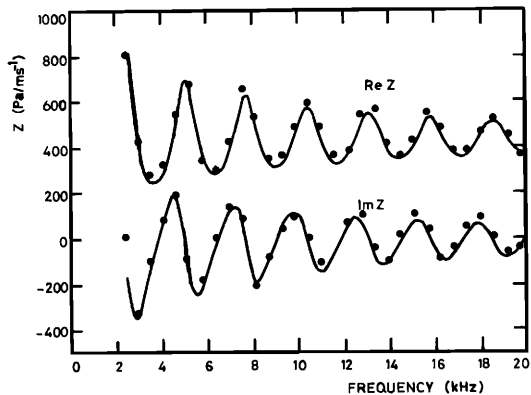


FIG. 6. The impedance  $Z$  of the layer of reticulated foam having a thickness equal to 6.15 cm at normal incidence. Prediction —; measurement ●●●.

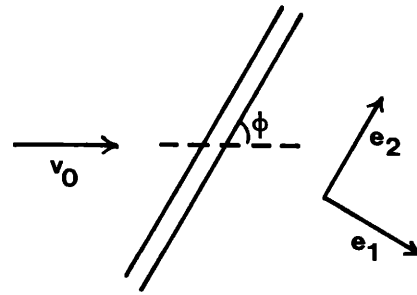


FIG. A1. A fiber in a planar plane, making an angle  $\phi$  with the mean direction of the static flow.

## V. CONCLUSION

For the glass wool studied, the characteristic dimensions can be calculated, and the surface impedance can be calculated with no adjustable parameter. The predicted and the measured surface impedance are very similar. For the reticulated foam, the characteristic dimensions are evaluated from a comparison between measured and predicted impedances. In previous models, adjustable parameters with no precise physical meaning were used. The concept of characteristic dimension gives a new meaning to these parameters.

## APPENDIX: EVALUATION OF THE VISCOUS CHARACTERISTIC DIMENSION IN PLANAR DIRECTIONS

A fiber in a planar wave, making an angle  $\phi$  with the mean velocity  $v_0$  of the static flow of inviscid fluid, is represented in Fig. A1.

The static flow can be considered as a superposition of two flows perpendicular and parallel to the direction of the fiber:

$$v_0 = v_0 l_1 \sin \phi + v_0 l_2 \cos \phi, \quad (A1)$$

where  $l_1$  and  $l_2$  are the unit vectors perpendicular and parallel to the fiber. In the same way, Eq. (1) becomes

$$\frac{2}{\Lambda_p} = \frac{\langle \int v_1^2 dS + \int v_2^2 dS \rangle}{\int v^2 dV}, \quad (A2)$$

where  $v_1$  and  $v_2$  are the velocity fields induced by  $v_0 l_1 \sin \phi$  and  $v_0 l_2 \cos \phi$ , and the symbol  $\langle \rangle$  denotes an average over  $\phi$  in the planar plane. Equation (A2) can be rewritten:

$$\frac{2}{\Lambda_p} = \left\langle \frac{\sin^2 \phi}{\Lambda_N} + \frac{\cos^2 \phi}{\Lambda'_N} \right\rangle = \frac{2 \times 3}{4\Lambda_N}. \quad (A3)$$

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