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► **To cite this version:**

Jean Allard, Bruno Brouard, Denis Lafarge, Walter Lauriks. Time reversal and reciprocity in sound propagation through elastic and porous layers. *Applied Physics Letters*, 1992, 61 (10), pp.1166-1167. 10.1063/1.107634 . hal-02518324

HAL Id: hal-02518324

<https://univ-lemans.hal.science/hal-02518324>

Submitted on 28 Nov 2023

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Citation: [Applied Physics Letters](#) **61**, 1166 (1992); doi: 10.1063/1.107634

View online: <http://dx.doi.org/10.1063/1.107634>

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Time reversal and reciprocity in sound propagation through elastic and porous layers

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(Received 25 March 1992; accepted for publication 22 June 1992)

Transmission of plane sound waves through elastic or porous layers involves longitudinal and transverse waves and can be described by equivalent multiport circuits. Different ports on each face of the layer are related to the normal and the tangential strains and velocities. The form of the impedance matrix of the circuits indicates that the coupling between normal and tangential conjugate variables is antireciprocal. This is due to the fact that the angle of incidence changes sign under time reversal.

A simple electrical two-port system as shown in Fig. 1 can be described by the following set of equations.

$$v_1 = A_{11}i_1 + A_{12}i_2, \quad v_2 = A_{21}i_1 + A_{22}i_2, \quad (1)$$

or

$$i_1 = \alpha_{11}v_1 + \alpha_{12}v_2, \quad i_2 = \alpha_{21}v_1 + \alpha_{22}v_2, \quad (2)$$

where the coefficients A_{ij} define an impedance matrix and the coefficients α_{ij} an admittance matrix. If the system is a network of inductances, resistances, and capacitors

$$\alpha_{12} = \alpha_{21}, \quad A_{12} = A_{21}, \quad (3)$$

and the network is reciprocal.

If a permanent magnetic field \mathbf{H} is present, e.g., if the two ports are coupled by Hall effect, Casimir¹ and Lampton² have shown that Eqs. (3) become

$$\alpha_{12}(\mathbf{H}) = \alpha_{21}(-\mathbf{H}), \quad A_{12}(\mathbf{H}) = A_{21}(-\mathbf{H}). \quad (4)$$

The minus sign is due to the fact that reciprocity is related to time reversal and \mathbf{H} is inverted under time reversal. The consequence of Eqs. (4) is that for the Hall effect coupler, Eqs. (3) can be rewritten

$$\alpha_{12} = -\alpha_{21}, \quad A_{12} = -A_{21}. \quad (5)$$

The system is antireciprocal.

A simple antireciprocal acoustic system is shown in Fig. 2. A plane wave impinges upon a layer of elastic solid at an angle of incidence θ . The rear face of the layer is bonded onto a rigid backing, making the x_1 and x_3 components of velocity equal to zero at the rear face. The equivalent circuit is represented in Fig. 2 where the components of the stress tensor and the velocity at A , at the front face of the layer, are denoted by σ_{33} , σ_{13} , v_3 , v_1 . Let $\mathbf{V}(x_3)$ be the vector:

$$\mathbf{V}(x_3) = [\sigma_{33}(x_3), \sigma_{13}(x_3), v_3(x_3), v_1(x_3)]^T. \quad (6)$$

The transfer matrix $[T(l)]$ relating the vectors \mathbf{V} at the two faces of an elastic layer of thickness l for an angle of incidence θ in air was calculated by Folds and Loggins.³ With the aid of the relation

$$\mathbf{V}[x_3(A)] = \{T[x_3(A) - x_3(B)]\}\mathbf{V}[x_3(B)]. \quad (7)$$

The matrix elements α_{12} , α_{21} , A_{12} , and A_{21} of the equivalent circuit of Fig. 2 are easily calculated from the elements T_{ij} of $T(l)$. For instance, α_{12} is given by

$$\alpha_{12} = -(T_{32}T_{44} - T_{34}T_{42}) / (T_{33}T_{44} - T_{34}T_{43}). \quad (8)$$

These elements are related by Eqs. (5), and the system is antireciprocal. Antireciprocal behavior can be demonstrated without calculation by noticing that $\sigma_{13}(A)$, $v_1(A)$ change sign with θ , but $\sigma_{33}(A)$, $v_3(A)$ are invariant, and that θ changes sign under time reversal.

Antireciprocal behavior is also encountered in the case of sound transmission through a stratified medium, as noted by Rudgers.⁴ Again, Eq. (7) can be applied and an impedance matrix calculated from the transfer matrix $[T]$. This matrix relates the stress components to the velocity components at the two faces of an elastic layer by

$$\begin{bmatrix} \sigma_{33}(A) \\ \sigma_{13}(A) \\ \sigma_{33}(B) \\ \sigma_{13}(B) \end{bmatrix} = [Z] \begin{bmatrix} v_3(A) \\ v_1(A) \\ v_3(B) \\ v_1(B) \end{bmatrix}. \quad (9)$$

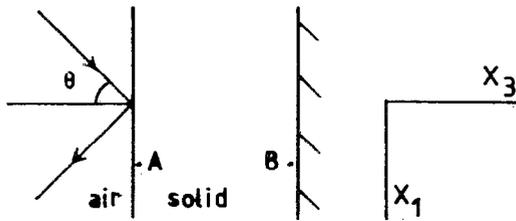
The matrix $[Z]$ presents the following structure

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ -Z_{12} & Z_{22} & Z_{23} & Z_{24} \\ -Z_{13} & Z_{23} & Z_{33} & Z_{34} \\ Z_{14} & -Z_{24} & -Z_{34} & Z_{44} \end{bmatrix}. \quad (10)$$

Other symmetries exist, but they are not related to reciprocity. The equivalent circuit is now a four-port cir-



FIG. 1. An electrical two-port system.



$$\begin{bmatrix} \sigma_{33}^f(A) \\ \sigma_{33}^s(A) \\ \sigma_{13}^s(A) \\ \sigma_{33}^f(B) \\ \sigma_{33}^s(B) \\ \sigma_{13}^s(B) \end{bmatrix} = [Z] \begin{bmatrix} v_3^f(A) \\ v_3^s(A) \\ v_1^s(A) \\ v_3^f(B) \\ v_3^s(B) \\ v_1^s(B) \end{bmatrix} \quad (11)$$



FIG. 2. A layer of elastic material bonded onto a rigid backing, in a plane acoustic field, and the equivalent electrical system related to stress and velocity components at A.

The elements Z_{ij} have been calculated numerically for different materials from the transfer matrix $[T]$ given by Allard *et al.*⁶ The matrix $[Z]$ always has the following structure

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} & Z_{16} \\ Z_{12} & Z_{22} & Z_{23} & Z_{24} & Z_{25} & Z_{26} \\ -Z_{13} & -Z_{23} & Z_{33} & Z_{34} & Z_{35} & Z_{36} \\ -Z_{14} & -Z_{24} & Z_{34} & Z_{44} & Z_{45} & Z_{46} \\ -Z_{15} & -Z_{25} & Z_{35} & Z_{45} & Z_{55} & Z_{56} \\ Z_{16} & Z_{26} & -Z_{36} & -Z_{46} & -Z_{56} & Z_{66} \end{bmatrix} \quad (12)$$

cuit and the description of the system is more involved. The relations $Z_{12} = -Z_{21}$, $Z_{34} = -Z_{43}$, correspond to the antireciprocal behavior described previously. The relations $Z_{13} = -Z_{31}$, $Z_{24} = -Z_{42}$, correspond to the fact that the same axes are used at both faces of the layer, and that identical couples (σ, v) at both faces correspond to opposite incoming energies from the external medium. The relations $Z_{14} = Z_{41}$ and $Z_{23} = Z_{32}$ are a consequence of the two successive changes of sign.

The elements are symmetric when related to two ports of the same type (normal stress in solid or fluid) at the same face, or two ports of different types at opposite faces. Otherwise they are antisymmetric. Biot theory gives impedance matrices which have the same structure as for the case of elastic solids.

In the context of the Biot theory,⁵ Allard *et al.*⁶ derived a similar representation of propagation in porous layers to that of Folds and Loggins for elastic layers. The couple (σ_{33}, v_3) splits into two couples (σ_{33}^s, v_3^s) and (σ_{33}^f, v_3^f) related to the frame (subscript s) and the fluid (subscript f) of the porous material, and the couple (σ_{13}, v_3) exists only for the frame. The matrix impedance $[Z]$ is now a 6×6 matrix:

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