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► **To cite this version:**

Yosra Dammak, Jean-Hugh Thomas, Mohamed Hèdi Ben Ghozlen. Lamb wave Propagation in Functionally Graded Piezoelectric Material Created by Internal Temperature Gradient. ICU 2015 International Congress on Ultrasonics, May 2015, Metz, France. pp.122-125, 10.1016/j.phpro.2015.08.057 . hal-02514161

**HAL Id: hal-02514161**

**<https://univ-lemans.hal.science/hal-02514161>**

Submitted on 21 Mar 2020

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2015 International Congress on Ultrasonics, 2015 ICU Metz

## Lamb wave Propagation in Functionally Graded Piezoelectric Material Created by Internal Temperature Gradient

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### Abstract

This work presents a theoretical study of the propagation behavior of lamb wave in a functionally graded piezoelectric material (FGPM). The piezoelectric material is polarized when the six fold symmetry axis is put along the propagation direction  $x_1$  and the material properties change gradually perpendicularly to the plate. The FGPM behavior is created by forming a temperature variation across the plate. The ordinary differential equation (ODE) and the Stiffness Matrix Method (SMM) are used to investigate the propagation of the lowest-order symmetric ( $S_0$ ) and antisymmetric ( $A_0$ ) Lamb wave modes.

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Peer-review under responsibility of the Scientific Committee of ICU 2015

*Keywords:* Lamb Wave; FGPM; Stiffness Matrix Method

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### 1. Introduction

The research of surface acoustic wave propagation problems related to FGPM have been widely studied. Some works are mainly based on analytical approaches as WKB (Wentzel-Kramers-Brillouin) method which is used to solve the coupled electromechanical field differential equation of love wave propagation in FGPM materials. The wave propagation model utilizes a unique gradient coefficient meaning that the electrical and mechanical parameters vary in the same way in term of thickness [1]. Different numerical procedures have been also used as Hybrid and Stiffness matrix concerning propagation of Saw in multilayered and FGPM systems. These methods make necessary the discretization of the graded structure into sublayers of averaged properties in the direction of the compositional

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gradient [2], and the constants distribution are unrealistic, they are all distributed with a defined mathematical function (linear, exponential, Gaussian or quadratic) [3], because it is difficult to estimate material property changes synchronously in terms of a certain law [4].

## 2. Theoretical background

### 2.1. Problem Statement

In the present work, the study is based on the differential equations associated with a FGPM plate written according the Thomson-Haskell parameterization of the Stroh formalism. The Stiffness matrix method is employed for solving the problem of Lamb wave in a FGPM plate of Lead Zirconate Titanate Ceramic (PZT-4D), with thickness labeled  $h$ , equal to 1,6 mm. The inhomogeneity is created by forming a temperature gradient along the plate (the  $x_3$  axis) and each material constants, piezoelectric, dielectric, and elastic has a different variation's profile. The  $(x_1, x_2, x_3)$  is the rectangular Cartesian coordinates and the  $(x_1, x_2)$  plane is parallel to lamb wave polarization, and the PZT six fold symmetry axis is put parallel to the propagation direction  $x_1$ , so that the Lamb mode becomes piezoactive.

### 2.2. Governing differential equations

The system of governing equations of the piezoelectricity, in the absence of body force and free charges, are written below [5]:

$$\rho \ddot{u}_i = \tau_{ij,j} , \quad D_{i,i} = 0 \quad (i=1, 3) \tag{1}$$

$$\tau_{ij} = c_{ijkl} u_{k,l} + e_{ij} \phi_{,l} , \quad D_i = e_{ikl} u_{k,l} - \epsilon_{il} \phi_{,l} \quad (i=1, 3) \tag{2}$$

Where  $u_j$  are the mechanical displacement components,  $\phi$  the electrical potential function, and  $\rho$  is the density. In the same way  $\tau_{ij}$  and  $D_j$  denote the stress and electric displacement field respectively. The remaining magnitudes  $c_{ijkl}$ ,  $e_{jk}$ ,  $\epsilon_{kl}$  are the elastic, piezoelectric and the electric constants, respectively. To describe the Lamb wave in FGPM, the following boundary and continuous conditions should be satisfied shorted and open circuit together with the mechanical free surface condition on both sides ( $x_3= 0, -h$ ):

Shorted circuit:  $\phi = 0$  and  $\tau_{i3} = 0$  Open circuit:  $\sigma = 0, \tau_{i3} = 0$

$\sigma$  refers to the charge density, it can be deduced from the normal electric displacement component values in the neighborhood of the plate free surface:

$$\sigma(-h) = D_3(-h^+) - D_3(-h^-) \tag{3}$$

$$\sigma(0) = D_3(0^+) - D_3(0^-) \tag{4}$$

We consider the problem of plane wave propagating along the  $x_1$  axis of the form  $\xi(x_3)\exp[j(k_1x_1-\omega t)]$  where  $k_1$  is the horizontal wave vector component,  $\omega$  is the angular frequency.  $\xi$  is the state vectors defined as:

$$\xi = [U, T]^T , U = (-j\omega u_i, -j\omega \phi) \text{ and } T = (\tau_{i3}, D_3).$$

The eight-component state vector for FGPM is utilized to write constitutive equations and motion equation under Thomson-Haskell parameterization of the Stroh formalism [1]:

$$\frac{d}{dx_3} \xi(x_3) = Q(x_3) \xi(x_3) \tag{5}$$

The  $Q(x_3)$ , fundamental acoustic tensor (6×6), is a function of material properties, wave number and frequency.

2.3. Implementation for a functionally graded piezoelectric material

In this study, all the material properties vary proportionally to the temperature variation created in the thickness direction of the piezoelectric plate in the range from 0°C to 100°C (Fig.1). Through this plate, constants distributions are realistic [6] and it would allow us to comprehend the influence of temperature gradient upon the characteristics of wave propagation. The variation of the density in the FGPM plate is so slight that it can be disregarded [7].

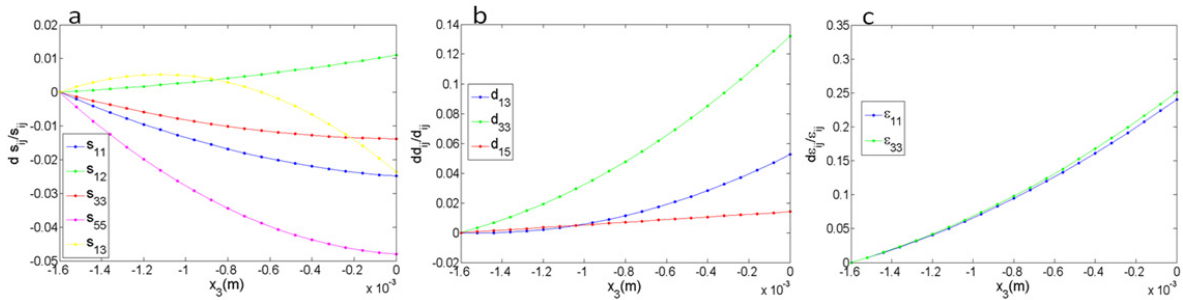


Fig. 1 Relative distribution model of the elastic (a), piezoelectric (b) and dielectric (c) constants.

3. Results

3.1. Effect of the gradient on the phase velocity

Figure 2 (a, b, c, d) show the effect of gradient on phase velocity as function of non-dimensional wave number  $h/\lambda$ . It can be seen that the phase velocity of  $S_0$  Lamb mode starts at a high value, and when the frequency increases, the phase velocity decreases and becomes near  $V_R$  (the non-dispersive Rayleigh velocity). The  $A_0$  mode performs differently, the phase velocity starts from zero value and becomes close to the same value  $V_R$  as the frequency increases. The phase velocity value of the lowest-order symmetric ( $S_0$ ) and anti-symmetric ( $A_0$ ) Lamb wave for the electrically open and shorted cases are also obtained denoted  $V_{oc}$  and  $V_{sc}$  respectively. The temperature gradient does not change the shape of the curves but introduces a jump in the dispersion curves compared to homogenous results; this variation is more significant in the symmetric mode (Fig. 2.a.b).

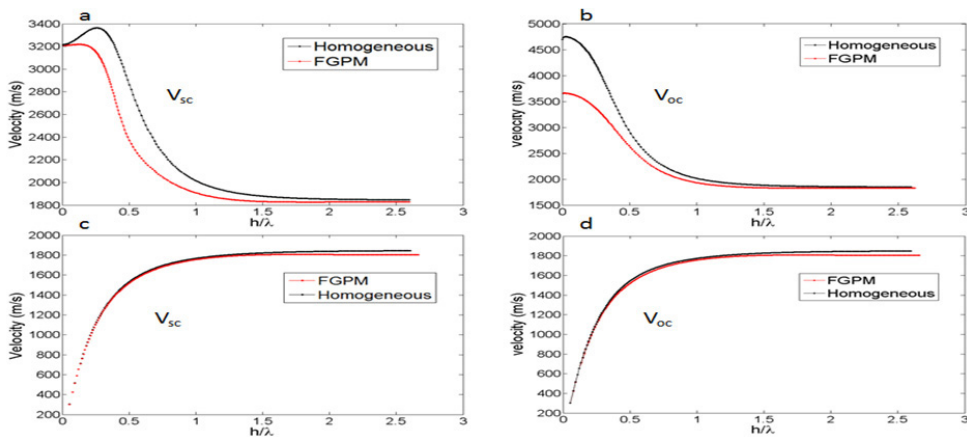


Fig. 2 Dispersion curves of  $S_0$  mode (a) shorted circuit (b) open circuit, and  $A_0$  mode (c) shorted circuit (d) open-circuit.

### 3.2. Effect of the gradient on the electromechanical coupling factor

For surface acoustic waves devices, a high electromechanical coupling factor of the wave is expected in engineering applications, it is defined as follows [9]:

$$k^2 = \frac{2(V_{oc} - V_{sc})}{V_{oc}} \quad (6)$$

The electromechanical coupling factor is presented in Fig.3. It can be found that the electromechanical coupling factor of the  $S_0$  decreases with 64% in the FGPM case, same for the  $A_0$  mode, with 88%.

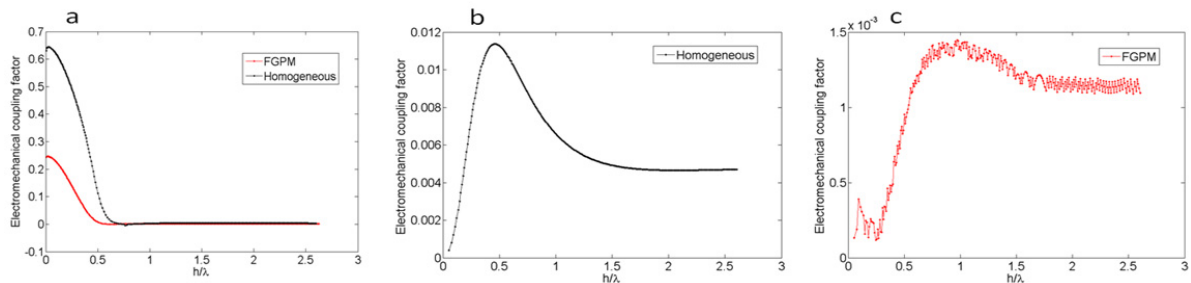


Fig. 3 Electromechanical coupling factor for  $S_0$  mode (a) and for  $A_0$  mode (b), (c).

## 4. Conclusion

The propagation of lamb wave in functionally graded piezoelectric material (FGPM) layer is investigated by a numerical matrix method. The coupled electromechanical field equations are solved exactly for the dispersion curves under both electrically open and shorted conditions. The numerical approach exhibits a high stability and a good convergence with the stratification of the plate. The effect of the temperature gradient on phase velocity and electromechanical coupling factor are plotted and discussed, the phase velocity of  $S_0$  mode decreases enormously compared to  $A_0$  mode in the FGPM case. In addition, the electromechanical coupling factor has a significant variation due to internal temperature gradient.

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