



## Vibroacoustic response sensitivity due to relative alignment of two anisotropic poro-elastic layers

Eleonora Lind Lind Nordgren, Peter Göransson, Jean-François Deü, Olivier Dazel

### ► To cite this version:

Eleonora Lind Lind Nordgren, Peter Göransson, Jean-François Deü, Olivier Dazel. Vibroacoustic response sensitivity due to relative alignment of two anisotropic poro-elastic layers. Journal of the Acoustical Society of America, 2013, 133 (5), pp.EL426-EL430. 10.1121/1.4802184 . hal-02458843

**HAL Id: hal-02458843**

**<https://univ-lemans.hal.science/hal-02458843>**

Submitted on 12 Jan 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Vibroacoustic response sensitivity due to relative alignment of two anisotropic poro-elastic layers

**Eleonora Lind Nordgren and Peter Göransson<sup>a)</sup>**

*Centre for ECO<sup>2</sup> Vehicle Design, MWL, KTH Royal Institute of Technology, Stockholm, Sweden*

*eleonora@kth.se, pege@kth.se*

**Jean-François Deü**

*Laboratoire de Mécanique des Structures et des Systèmes Couplés (LMSSC), Conservatoire National des Arts et Métiers, Paris, France*

*jean-francois.deu@cnam.fr*

**Olivier Dazel**

*Laboratoire d'Acoustique de l'Université du Maine - UMR CNRS 6613, Le Mans, France*

*olivier.dazel@univ-lemans.fr*

**Abstract:** The effects of relative alignment of two different types of an-isotropic open cell porous materials are investigated in terms of the acoustic response of a multi-layered configuration. Numerical experiments, where gradient based optimization techniques were used, are conducted to find possible extremal values. It is shown that, depending on the degree of anisotropy of the porous material properties, their angular orientations have a significant and frequency dependent influence on the measured response. The results highlight the importance of further advancing the knowledge of anisotropic porous material behavior.

## 1. Introduction

The focus of the present contribution is to explore the influence on the vibroacoustic response of anisotropy in open cell porous materials in general and the effects of aligning two layers of an identical material relative to each other in particular. The sensitivity to relative alignment is studied in terms of the extremal values to an optimization problem with the relative orientation angles as variables. While a completely general material model would imply that the elastic, the acoustic, the anelastic, and the visco-acoustic material tensors all have their own material coordinate system, it is here assumed for simplicity and transparency that all properties are given in the same reference coordinate system. The relative alignment of the two layers is then constructed as rotations of the material reference systems, with respect to the global body coordinate axes. A complete description of the theoretical background and the models used will not be given here, but the interested reader is referred to the literature cited throughout.

The starting point of the current work is the general modeling paradigm set up by Biot and in particular its extension to anisotropic porous materials.<sup>1</sup> Here a mixed displacement-pressure formulation, recently proposed by Hörlin and Göransson<sup>2</sup> as a generalization for anisotropic materials of the work by Atalla *et al.*<sup>3</sup> is used. This model incorporates anisotropy of both the frame, elastic constants, and permeability, acoustic coefficients. For the acoustic modeling of the porous material behavior, the Biot-Johnson-Champoux-Allard model is used.<sup>4</sup>

---

<sup>a)</sup>Author to whom correspondence should be addressed.

There is a general awareness that anisotropy may have a significant influence on the acoustic behavior of porous materials.<sup>5</sup> It is also well established that some of the parameters used in the Biot-Johnson-Champoux-Allard model differ in different directions in anisotropic materials.<sup>6</sup> In this first study of the effects of the relative alignment between two layers, the sensitivity analysis has been limited to three directionally dependent tensors, namely, the static flow resistivity tensor  $\sigma_{ij}$ , the tortuosity tensor  $\alpha_{ij}$ , and the Hooke's matrix for the solid frame bulk moduli  $C_{ijkl}$ . All remaining parameters are assumed to be isotropic.

## 2. Properties of the anisotropic materials studied

Two different porous materials are studied, an orthotropic foam and a transversely isotropic fibrous wool, differing in their micro-structural build and in general exhibiting different material symmetries. The material properties of the fibrous wool are known from previous work,<sup>7</sup> density  $14.45 \text{ (kg/m}^3\text{)}$ , porosity  $0.994$ , viscous and thermal characteristic lengths  $6.25 \times 10^{-5} \text{ (m)}$  and  $12.5 \times 10^{-5} \text{ (m)}$ , respectively. The foam, however, is a hypothetical material, with density  $22.1 \text{ (kg/m}^3\text{)}$ , porosity  $0.98$ , viscous and thermal characteristic lengths  $1.1 \times 10^{-4} \text{ (m)}$  and  $7.4 \times 10^{-4} \text{ (m)}$ , respectively. The direction dependent elastic moduli of the foam and the fibrous wool are given, in the body coordinate (reference system), in Eqs. (1) and (2), in units of  $10^3 \text{ (Pa)}$ , respectively, as

$$\begin{aligned} C_{11}^{\text{foam}} &= 40; C_{12}^{\text{foam}} = 33; C_{13}^{\text{foam}} = 37; C_{22}^{\text{foam}} = 89; C_{23}^{\text{foam}} = 131; \\ C_{33}^{\text{foam}} &= 300; C_{44}^{\text{foam}} = 26; C_{55}^{\text{foam}} = 21; C_{66}^{\text{foam}} = 26; \end{aligned} \quad (1)$$

$$\begin{aligned} C_{11}^{\text{fibrous}} &= C_{22}^{\text{fibrous}} = 17; C_{33}^{\text{fibrous}} = 1; \\ C_{44}^{\text{fibrous}} &= C_{55}^{\text{fibrous}} = 2; C_{66}^{\text{fibrous}} = 14. \end{aligned} \quad (2)$$

For the flow resistivity tensor, both types of materials are taken as transversely isotropic and the actual values used, in  $10^3 \text{ (Pa s/m}^2\text{)}$ , are given by Eq. (3) for both the orthotropic foam and the fibrous material

$$\sigma_{11}^{\text{foam}} = \sigma_{12}^{\text{foam}} = 38; \sigma_{33}^{\text{foam}} = 55; \sigma_{11}^{\text{fibrous}} = \sigma_{12}^{\text{fibrous}} = 21; \sigma_{33}^{\text{fibrous}} = 41. \quad (3)$$

Note the difference in the relation between visco-acoustic and elastic properties between the two materials. The tortuosity used is given by Eq. (4) for both materials.

$$\alpha_{11}^{\text{foam}} = 1.5; \alpha_{12}^{\text{foam}} = 1.0; \alpha_{33}^{\text{foam}} = 2.0; \alpha_{11}^{\text{fibrous}} = \alpha_{12}^{\text{fibrous}} = \alpha_{33}^{\text{fibrous}} = 1.12. \quad (4)$$

## 3. Optimization problem

The response sensitivity due to variation of the alignment of two identical porous layers is analyzed through the solution of an optimization problem using previously published techniques.<sup>8</sup> In a similar setup, a finite sized panel geometry, including the effects of boundaries and interaction with a closed acoustic cavity at one surface, is used as a simulation model. A cost function based on the acoustic pressure multiplied with the frequency resolution,  $\Delta f_f$ , and summed over the entire frequency range to form the sound pressure level [SPL, see Eq. (5)], evaluated in a subvolume of the air cavity connected to the panel,

$$\langle \text{SPL}(\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2) \rangle_{\Omega_{\text{sub}}} = 10 \log \left( \sum_{f=f_1}^{f_{\text{max}}} \frac{1}{N} \sum_{n=1}^N p_{f_n}^2 \Delta f_f \right) / p_0^2, \quad (5)$$

where  $\alpha_i$ ,  $\beta_i$ , and  $\gamma_i$ ,  $i = 1, 2$  are the right hand rotations, i.e., the Euler angles with Z-Y-X fixed axis rotation bound between  $-\pi/2$  and  $\pi/2$ , for layers 1 and 2,

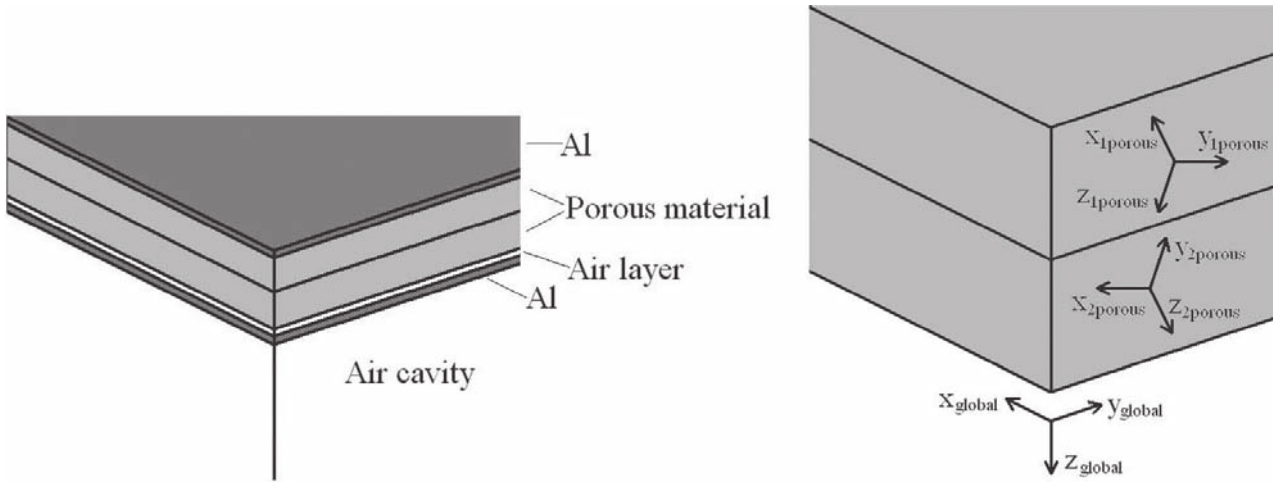


Fig. 1. Left: Layer configuration of the tested panels. Al(uminum) sheet thicknesses 0.001 m, porous layers 0.042 m each, and air layer 0.001 m. Right: Global and local co-ordinate axis systems together with an example of possible layer rotations of porous layers 1 and 2.

respectively. Maxima and minima of the applied cost function are computed, with variables being the unknown rotations, around a fixed coordinate system, of the constitutive parameters, see Fig. 1. The optimization problem was solved using the GCMMA optimizer by Svanberg.<sup>9</sup> As this is a gradient based algorithm, the required gradients were calculated using finite differencing.

#### 4. Simulation model

The numerical model is composed of a multi-layered flat panel facing an acoustic cavity of equal lateral dimension,  $0.5 \times 0.5$  m. The panel, see Fig. 1, has aluminum face sheets, both of 0.001 m thickness, and two layers of poro-elastic material, both of 0.042 m thickness. One of the face sheets is elastically bonded to the porous layers and has unit force excitation applied in the  $z$ -direction over the surface  $z = 0$  bound by  $x = 0.1$ ,  $x = 0.2$ ,  $y = 0.1$ , and  $y = 0.2$ . The opposite face sheet is separated from the porous layers by a thin air gap. Homogeneous natural boundary conditions are applied along the edges  $x = 0$ ,  $x = L_x$ ,  $y = 0$ , and  $y = L_y$ . The air cavity in which the acoustic response in Eq. (5) is evaluated is 1.4 m in the  $z$ -direction and has a subvolume centered in the  $x$ - $y$  plane, with dimensions  $0.3 \times 0.3 \times 0.3$  m and placed 0.2 m from the surface of the unbonded face sheet. The inner walls of the air cavity at  $x = 0$ ,  $y = 0$ , and  $z = L_z$  were assigned a non-frequency-dependent normal surface impedance of  $257 + 563i$ . The boundaries of the air cavity at  $x = L_x$  and  $y = L_y$  were considered to be acoustically hard.

#### 5. Results and discussion

For both types of materials, the total SPL difference between max and min solutions was found to be around 4.5 dB, see Table 1 which shows the maximum and the minimum solutions found. For the transversely isotropic, fibrous material, the main influence of aligning the material properties relative to each other occurs in the lower part of the studied frequency range, see the right part of Fig. 2. At the design point, where the angles correspond to the minimum solution, the response in the dominant peak is lowered in amplitude, shifted downwards in frequency, and the apparent damping is increased. For the remaining part of the frequency spectrum (not shown in the figure), the difference is small. For the anisotropic foam, the acoustic response computed with the angles corresponding to the minimum solution is reduced at the lower end of the frequency range studied, while the opposite holds for higher frequencies, suggesting that with a different frequency weighting another set of optimal angles would have been found.<sup>8</sup>

As previously has been discussed in Ref. 10, for the studied configuration the in-plane solid frame deformations are amplified in an orthotropic material as

Table 1. Absolute, i.e., Euler angles, and relative, i.e., direction cosines between the axes of the rotated material coordinate systems, rotations at maximum and minimum points. Top: Orthotropic foam. Bottom: Fibrous wool.

Euler angles extreme points			
Foam	Layer 1		Layer 2
Min	[13° 22° –18°]		[44° 41° –16°]
Max	[73° 61° 90°]		[–51° –90° 20°]
Direction cosines between rotated material coordinate systems			
	<i>x</i> -axis	<i>y</i> -axis	<i>z</i> -axis
Min	35°	33°	20°
Max	86°	90°	151°
Euler angles extreme points			
Fibrous	Layer 1		Layer 2
Min	[0° –32° –13°]		[0° 24° 26°]
Max	[0° 53° 28°]		[0° 53° –85°]
Direction cosines between rotated material coordinate systems			
	<i>x</i> -axis	<i>y</i> -axis	<i>z</i> -axis
Min	0°	0°	62°
Max	0°	0°	60°

compared to an equivalent isotropic. When the material coordinate system is rotated relative to the body coordinates, as is the case in the present work, shear coupling terms appear (this may be verified by computing the transformed Hooke's matrix using the rotation angles from Table 1). These shear coupling terms amplify the in-plane deformations further as well as tending to induce even shorter wavelengths in the anisotropic poro-elastic materials when compared to both orthotropic materials as well as their equivalent isotropic representations.

A preliminary explanation of the outcome of the present sensitivity study and the pronounced low frequency effects exhibited could be deduced using the above cited observations. In the configuration chosen for this study, an air gap is separating the porous layers from the face sheet interacting with the acoustic cavity, where the response measured by the cost function is evaluated. The shorter wavelengths in combination with the amplified in-plane deformation, induced by the anisotropies at the

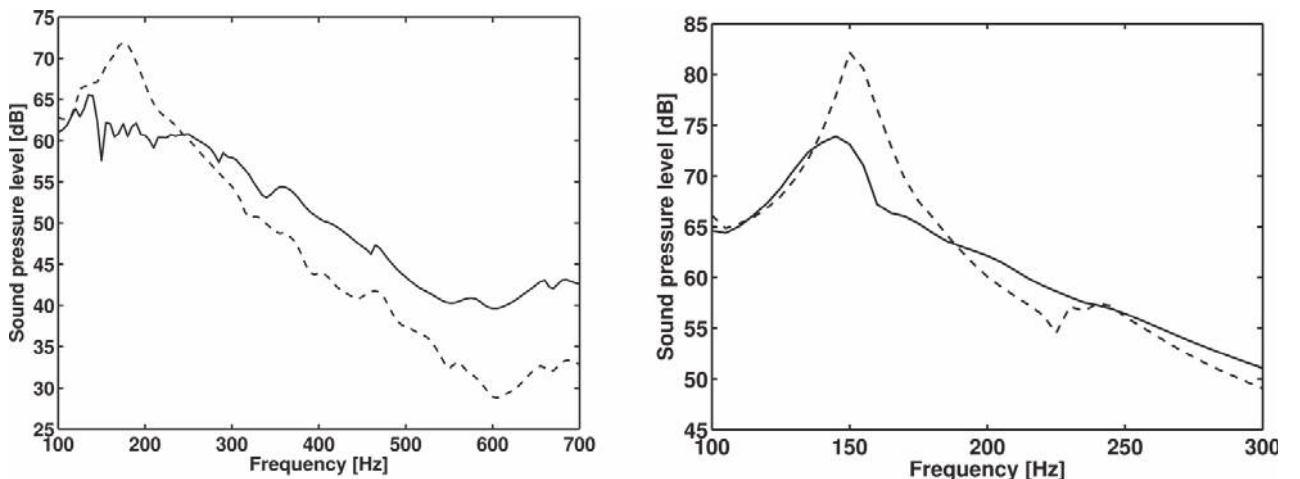


Fig. 2. SPL spectra computed for maximum and minimum solutions of the optimization problem. Left: Orthotropic foam. Right: Fibrous wool. Minimum solution = solid line; maximum solution = dashed line.

minimum configurations, could then explain a reduced coupling between the acoustic pressure field in the separating air gap, and the unbonded face sheet. This effect together with the global change in elastic stiffness, resulting from the rotation of the material directions, combine to a reduced system response.

## 6. Summary and conclusions

There appear to be significant possibilities of influencing the acoustic response of multi-layered panel configurations through proper orientation of anisotropic poroelastic material properties. However, to further explore this a deeper understanding of the anisotropic material properties, including the different mechanisms of energy losses, needs to be established.

## References and links

- <sup>1</sup>M. A. Biot, “Mechanics of deformation and acoustic propagation in porous media,” *J. Appl. Phys.* **33**, 1482–1498 (1962).
- <sup>2</sup>N. E. Hörlin and P. Göransson, “Weak, anisotropic symmetric formulations of Biot’s equations for vibro-acoustic modeling of porous elastic materials,” *Int. J. Numer. Methods Eng.* **84**, 1519–1540 (2010).
- <sup>3</sup>N. Atalla, M. A. Hamdi, and R. Panneton, “Enhanced weak integral formulation of the mixed (u,p) poroelastic equations,” *J. Acoust. Soc. Am.* **109**, 3065–3068 (2001).
- <sup>4</sup>J. Allard and N. Atalla, *Propagation of Sound in Porous Media: Modeling Sound Absorbing Materials* (John Wiley & Sons, New York, 2009).
- <sup>5</sup>P. Khurana, L. Boeckx, W. Lauriks, P. Leclaire, O. Dazel, and J.-F. Allard, “A description of transversely isotropic sound absorbing porous materials by transfer matrices,” *J. Acoust. Soc. Am.* **125**, 915–921 (2009).
- <sup>6</sup>S. Sahraoui, E. Mariez, and M. Etchessahar, “Linear elastic properties of anisotropic open-cell foams,” *J. Acoust. Soc. Am.* **110**, 635–637 (2001).
- <sup>7</sup>H. J. Rice and P. Göransson, “A dynamical model of light fibrous materials,” *Int. J. Mech. Sci.* **41**, 561–579 (1999).
- <sup>8</sup>E. Lind-Nordgren and P. Göransson, “Optimizing open porous foam for acoustical and vibrational performance,” *J. Sound Vib.* **329**, 753–767 (2010).
- <sup>9</sup>K. Svanberg, “A class of globally convergent optimization methods based on conservative convex separable approximations,” *SIAM J. Optim.* **12**, 555–573 (2002).
- <sup>10</sup>P. Göransson and N. E. Hörlin, “Vibro-acoustic modeling of anisotropic porous elastic materials: A preliminary study of the influence of anisotropy on the predicted performance in a multi-layer arrangement,” *Acta. Acust. Acust.* **96**, 258–265 (2010).